EVALUATION OF THE CONDITIONS OF VAPOR BUBBLE SEPARATION DURING NUCLEATE BOILING

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The quantities characterizing vapor bubble separation are evaluated. A quantitative description of the movement of a vapor bubble after separation is given.

One of the possible aspects of constructing an approximate theory of boiling is to find the relation between the microcharacteristics of boiling and its integral characteristics needed for engineering calculations. To accomplish this task we must first of all find the relation between the microcharacteristics of boiling (size of vapor bubbles, frequency of their separation from the heater, characteristic velocities of bubbles and their life time) and the physical properties of the medium and regime parameters of the process, including the acceleration of gravity.

Heretofore the bubble separation size was estimated by the Fritz equation [1]

$$D_d = 0.020\theta \sqrt{\frac{\sigma}{(\rho - \rho'')g}}, \quad (1)$$

which is obtained from an approximate solution of the problem of static stability of a bubble on a plane horizontal wall. Along with this, in finding the relation between the frequency of separation of bubbles and their diameter it was assumed that the bubble velocity at the instant of separation is equal to its buoyancy velocity in an unbounded liquid [1], although at the instant of quasi-static separation the bubble velocity should be equal to zero.

According to (1), the value of D_d does not depend on the mean temperature difference and in a wide range of pressures is practically independent of the saturation pressure, which contradicts the experimental data [2-5]. The observed change of D_d as a function of the acceleration of gravity, close to $D_d \sim g^{-1/3}$ [6, 7], also diverges from Eq. (1). Finally, in the case of boiling of cryogenic liquids for which the contact angle θ is close to zero an estimate of the bubble separation diameter by (1) loses all sense.

The known estimates of the value of D_d [8, 9], which were obtained from the balance of forces acting on the bubble at the instant of separation, lead in the limiting case to Eq. (1) without resolving the aforementioned contradictions. In these works consideration of the dynamic forces was approximate (with an accuracy to the unknown coefficient) and one-sided: either the drag force [8] or the inertial force [9] was considered.

In all the cited theoretical papers, just as in this one, the separation of individual, noninteracting bubbles was studied. Another approach is suggested in [10], where an ensemble of synchronously growing bubbles is considered. The equation for the bubble separation diameter obtained in [10] also leads to (1), but not at high, as in [8, 9], but at rather low pressures. The dynamic forces in the case of an ensemble of bubbles try to separate the bubble from the heater and their action is intensified with an increase of the number of vaporization centers, i.e., with an increase of pressure.

In the present article we propose to evaluate the conditions of separation of noninteracting vapor bubbles in which we will consider the surface tension force, inertial force, and drag force; the last two forces are evaluated with consideration of the effect of the wall [6, 7]. Unlike [8, 9] we will assume that in the limiting case of the quasi-static regime of separation of the bubbles the latter separate from microcavities.

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This assumption is substantiated at least for cryogenic liquids and leads to the dependence of D_d on g coinciding with the experimental [6, 7].

1. Evaluation of the Quantities Characterizing the Start of the Stage of Vapor Bubble Separation. We will use a scheme of motion of the vapor bubble from the instant of its occurrence ($\tau = 0$) until separation ($\tau = \tau_d$) similar to that proposed in [11]. We will distinguish three stages of bubble motion near the horizontal heater (wall).

Stage I, the growth stage, occupies the time interval from the occurrence of the bubble ($\tau = 0$) until the start of its separation from the wall ($\tau = \tau_0$). During stage I the forces acting on the vapor bubble toward the wall (downward) are greater than the forces trying to separate the bubble from the wall. The center of gravity of the spherical bubbles moves according to the law of change of its radius

$$S = R = \beta\tau^{\frac{1}{2}}. \tag{2}$$

Stage II, the separation stage, begins at instant $\tau = \tau_0$, at which the forces acting on the vapor bubble contacting the wall arrive at equilibrium. The vapor bubble continues to expand, in this case $S > R$, $\dot{S} > \dot{R}$. From the balance of forces acting on the vapor bubble during stage II we can obtain its equation of motion. Stage II ends with separation of the bubble from the "foot" or with an abrupt departure from the wall with a simultaneous cessation of the growth of the vapor bubble. The coordinate of the position of the center of gravity of the bubble at the instant of separation $\tau = \tau_d$ can be found in the first approximation from the empirical relationship [12]

$$S_d \approx 1.5R_d. \tag{3}$$

During stage III, the buoyancy stage, the velocity of the bubble increases to a constant value u, determined by the balance of the buoyant force and drag.

We will determine the conditions of transition of stage I to stage II, i.e., the start of bubble separation, from the equilibrium of forces

$$F_g = F_R + F_\nu + F_\sigma, \tag{4}$$

where F_g is the buoyant force, F_R is the reaction force of the liquid, F_ν is the drag, and F_σ is the surface tension. Using the expressions of the velocity potential for a bubble expanding near a wall [6, 7], method of determining F_ν [13], and (2), we obtain:

$$F_R = \frac{4}{21}\pi\rho R^2(8\ddot{R}R + 15\dot{R}^2) = \frac{\pi}{3}\rho\beta^4, \tag{5}$$

$$F_\nu = 20\pi\mu\dot{R}R = 10\pi\mu\beta^2, \tag{6}$$

$$F_\sigma = 2\pi R_c\sigma, \tag{7}$$

$$F_g = \frac{4}{3}\pi R^3(\rho - \rho'')g. \tag{8}$$

In calculating the surface tension we will take as R_c, for determinacy, the optimal radius of the cavity during nucleation [14]

$$R_c = \frac{4\sigma T_s}{L\rho''\Delta T}. \tag{9}$$

Substituting (5)-(8) into (4), we obtain the values of the time and radius of the bubble corresponding to the start of its separation:

$$\tau_0 = C_0\beta^{\frac{2}{3}}g^{-\frac{2}{3}}, \quad R_0 = C_0^{\frac{1}{2}}\beta^{\frac{4}{3}}g^{-\frac{1}{3}}, \tag{10}$$

where

$$C_0 = \frac{1}{2\sqrt[3]{2}}\left(1 + \frac{F_\nu}{F_R} + \frac{F_\sigma}{F_R}\right)^{\frac{2}{3}}\left(\frac{\rho}{\rho-\rho''}\right)^{\frac{2}{3}} = \frac{1}{2\sqrt[3]{2}}\left(1 + \frac{30\nu}{\beta^2} + \frac{6R_c\sigma}{\rho\beta^4}\right)^{\frac{2}{3}}\left(\frac{\rho}{\rho-\rho''}\right)^{\frac{2}{3}}. \tag{11}$$

2. Evaluation of the Quantities Characterizing the Start of the Stage of Buoyancy of the Vapor Bubbles. The equation of motion of the bubble in stage II can be obtained from Newton's second law

$$\frac{d}{d\tau}(M''\dot{S}) = F_R + F_D - F_g, \tag{12}$$

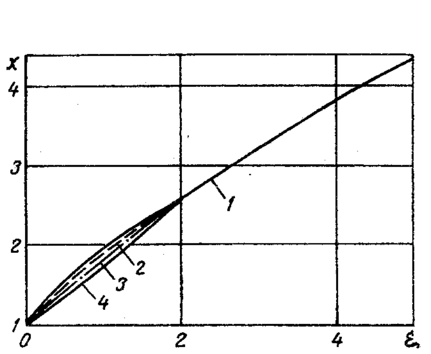

Fig. 1

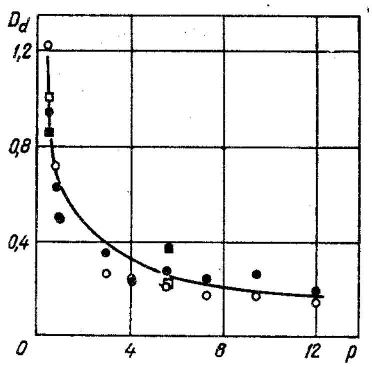

Fig. 2

Fig. 1. Dimensionless time of separation of vapor bubbles $x = \tau_d / \tau_0$ vs parameters ξ and γ: 1) $\gamma = 0$; 2) 1; 3) 3; 4) 10.

Fig. 2. Bubble separation diameter D_d (mm) vs pressure p (bar). Closed points: experimental data; open points: calculation.

where F_D is the drag for a bubble not contacting the wall [6, 7]:

$$F_D = 12\pi\mu R\dot{S}. \tag{13}$$

The equation of bubble motion which is obtained after substituting (8) and (13) and the expressions for F_R [6, 7] into (12) with disregarding of the left side of (12) in view of its smallness is easily solved by the numerical method [6]. As a first approximation we can use the equation of motion of an expanding bubble in an unbounded liquid

$$\ddot{S}R + 3\dot{S}\dot{R} + \frac{18\nu}{R}\dot{S} = 2gR. \tag{14}$$

Substituting the law of bubble growth (2) into (14) and solving (14) for initial conditions $S(\tau_0) = R_0 = \beta\tau_0^{1/2}$, $\dot{S}(\tau_0) = 0.5\beta\tau_0^{-1/2}$, we will have

$$S = \frac{g\tau^2}{2.5+\gamma} - \frac{1}{1+2\gamma}\left(\beta\tau_0^{1+\gamma} - \frac{g}{2.5+\gamma}\tau_0^{\frac{5}{2}+\gamma}\right)\tau^{-\frac{1}{2}-\gamma} - \frac{2g\tau_0^2}{2.5+\gamma} + 2\,\frac{1+\gamma}{1+2\gamma}\,\beta\tau_0^{\frac{1}{2}}, \tag{15}$$

where $\gamma = 18\nu/\beta^2$.

From (3) and (15) we obtain the equation for $x = \tau_d/\tau_0$, the solution of which gives the time of separation τ_d:

$$x^2 - 3\xi x^{\frac{1}{2}} - \frac{2(\xi-2)}{1+2\gamma}x^{-\frac{1}{2}-\gamma} - \frac{5+2\gamma}{1+2\gamma} + \frac{4(1+\gamma)}{1+2\gamma}\xi = 0. \tag{16}$$

The solution of (16) is determined by the value of the parameter $\xi = (2.5+\gamma)/2C_0^{3/2}$ and depends weakly on the parameter γ (see Fig. 1). In the first approximation the dependence of x on ξ can be represented in the form

$$x = 1 + 0.7\xi. \tag{17}$$

We note that on including in Eq. (16) the term related with surface tension (7) its solution remains practically the same.

The bubble separation radius and the time of its separation can be represented in the form

$$R_d = C_R\beta^{\frac{4}{3}}g^{-\frac{1}{3}}, \quad \tau_d = C_R^2\beta^{2/3}g^{-\frac{2}{3}}, \tag{18}$$

where $C_R = (C_0 x)^{1/2}$.

3. Dynamic and Quasi-Static Bubble Separation Regimes. We will consider two limiting bubble separation regimes: quasi-static for $F_\sigma \gg F_R$, $F_\sigma \gg F_\nu$ and dynamic for $F_R \gg F_\sigma$, $F_R \gg F_\nu$. The first regime is characteristic for relatively high pressures (for example, for $p \gg 1$ bar in the case of such liquids as water and alcohols), and the second for low ($p \ll 1$ bar). In the quasi-static regime the equation of bubble motion degenerates to the condition of bubble separation at $\tau = \tau_0$; from Eq. (16) follows $\tau_d \approx \tau_0$. The conditions of bubble separation in this case are written in the form

TABLE 1. Value of the Growth Moduli β during Boiling of Water

ΔT, deg	Ja	β, cm/sec$^{1/2}$		
		after Labuntsov	after Cole–Shulman	after Plesset–Zwick
10	30	0,8	1,3	2,4
20	60	1,1	2,2	4,8

$$R_d = \left[\frac{3}{2}\cdot\frac{R_c\sigma}{g(\rho-\rho'')}\right]^{\frac{1}{3}},\quad \tau_d = \beta^{-2}\left[\frac{3}{2}\cdot\frac{R_c\sigma}{g(\rho-\rho'')}\right]^{\frac{2}{3}}. \tag{19}$$

In the dynamic regime of bubble separation $\xi = 5$, which gives $x \approx 4.5$, and the separation conditions have the form

$$R_d \approx 1.34\beta^{\frac{4}{3}} g^{-\frac{1}{3}},\quad \tau_d \approx 1.8\beta^{\frac{2}{3}} g^{-\frac{2}{3}}. \tag{20}$$

An equation for R_d, analogous to (19), was proposed by Zuber [5]; Eq. (20) with the same numerical coefficient was obtained by Labuntsov and Yagov [15, 16] on the basis of a different model of separation.

The conditions of existence of the dynamic regime are determined from (18) by the inequality $F_R > F_\sigma$, or

$$\beta > \sqrt{\frac{6R_c\sigma}{\rho}}. \tag{21}$$

For water at atmospheric pressure, from (21) follows $\beta > 0.7$ cm/sec$^{1/2}$ at $\Delta T = 10°$ and $\beta > 0.6$ cm/sec$^{1/2}$ at $\Delta T = 20°$. The bubble growth modulus β can be determined by the equation

$$\beta = C_\beta \mathrm{Ja}^{n_\beta}\sqrt{a}, \tag{22}$$

where $n_\beta = 0.5$, $C_\beta = \sqrt{12}$ (Labuntsov equation) if Ja < 10–20; $n_\beta = 0.75$, $C_\beta = 2.5$ (Cole–Shulman equation) if Ja ≥ 20–30 [15]. For boiling of superheated liquids β is described by the Plesset–Zwick equation: $n_\beta = 1$, $C_\beta = 1.95$. Table 1 presents the values of β for water calculated by (22).

It follows from Table 1 that the bubble separation regime in the case of boiling of water at atmospheric pressure can be considered dynamic at $\Delta T \geq 10°$.

The equations for τ_d and R_d permit a qualitative explanation of the contradictory statements in the literature on the dependence of the frequency and separation diameter on the density of the heat flux q and pressure p. Suppose that $q \sim \Delta T^3$. Then, using the Labuntsov dependence for β, in the case of the quasi-static regime we obtain

$$R_{dst} \sim \Delta T^{-\frac{1}{3}} \sim q^{-\frac{1}{9}},\quad \tau_{dst} \sim \Delta T^{-\frac{5}{3}} \sim q^{-\frac{5}{9}},$$

and in the dynamic regime

$$R_{d\mathrm{dyn}} \sim \Delta T^{\frac{2}{3}} \sim q^{\frac{2}{9}},\quad \tau_{d\mathrm{dyn}} \sim \Delta T^{-\frac{1}{3}} \sim q^{-\frac{1}{9}}.$$

Thus an increase of q in the quasi-static bubble separation regime should lead to a noticeable reduction of the time of their growth τ_d at practically identical separation diameters D_d. In the dynamic regime an increase of q leads to a slight increase of D_d at a practically unchanged time of growth.

The regularities of the change of D_d and τ_d with pressure will also be different. Considering as the first approximation $R_c \sim p^{-1}$, $\beta \sim p^{-1/2}$, we obtain respectively for the quasi-static and dynamic regimes

$$R_{dst} \sim p^{-\frac{1}{3}},\quad \tau_{dst} \sim p^{\frac{1}{3}};\quad R_{d\mathrm{dyn}} \sim p^{-\frac{2}{3}},\quad \tau_{d\mathrm{dyn}} \sim p^{-\frac{1}{3}}.$$

Thus in the case of the quasi-static regime the separation diameter of the bubble decreases with pressure with a simultaneous increase of the bubble growth time.

Equations (18) determine the optimal values of R_d and τ_d. The lower limit of these values is determined by Eqs. (1), (11). To find the upper limit we use instead of (3) the condition $\dot{S}_d \leq u$, having determined the constant bubble buoyancy velocity as $u = 2.1\sqrt{R_d g}$ [13]. Having differentiated (15) with respect to time, substituted the value $\dot{S}_d = u$ into it, and transformed the equation obtained, we arrive at Eqs. (18),

TABLE 2. Comparison of Theoretical Separation Diameters of Vapor Bubbles and Experimental Data

p, bar	ΔT, deg	D_d, mm		
		experimental data [4]	calc. by (18)	calc. by [10]
11,95	2,55	0,2	0,145	0,115
9,5	3,2	0,27	0,18	0,18
7,35	3,4	0,252	0,175	0,225
5,65	4,35	0,28	0,21	0,25
5,69	5,1	0,368	0,22	0,20
4,22	2,8	0,248	0,255	0,82
3,16	5,5	0,352	0,265	0,50
0,97	4,9	0,512	0,515	0,95
0,9	9,3	0,657	0,72	0,96
0,588	16,6	0,932	1,20	0,90
0,578	12,2	0,83	0,99	0,98

but with different values of the coefficient C_R. In particular, for the dynamic separation regime we obtain $C_R \approx 1.9$, which exceeds only by 40% the optimal value $C_R = 1.34$.

In the case of the dynamic regime there is a simultaneous decrease of the separation diameter and life time of the bubble. If an increase of pressure causes transition from the dynamic to the quasi-static regime, the life time of the bubble can remain unchanged due to some mutual compensation of the dynamic forces and surface tension. However, the bubble separation diameter (for the same temperature difference) will always decrease with increase of pressure.

To check the correctness of the relations proposed, the separation diameters of vapor bubbles during boiling of Freon-12 were calculated. The results of the calculations and Danilova's experimental data [4] for the same temperature differences and pressures are presented in Fig. 2. The average experimental values [4] have the same scatter relative to the average experimental curve as the calculated values of D_d.

Table 2 presents the results of calculating D_d by Eq. (18) and by the equation obtained in [10] for an ensemble of simultaneously growing bubbles. The value of the contact angle in the latter case was assumed equal to 45° [4].

We see from Table 2 that in the majority of cases both theoretical equations with a maximum error of ±40% describe the experimental data. However, equation [10] gives markedly overstated values of D_d (for instance, by 230% for $p = 4.22$ bar) in the case of small temperature differences, i.e., in the case of a relatively small number of vaporization centers. In addition, equation [10] gives practically the same values of D_d at pressures below 1 bar, which is related with the inapplicability of the static model of bubble separation at low pressures.

4. Relation between Separation Diameter and Frequency of Separation of Vapor Bubbles. The relationships presented permit obtaining certain equations relating the frequency and diameter during bubble separation. We note that

$$f = \frac{1}{(\tau_d + \tau_w)} = \frac{1}{\tau_d} \cdot \frac{\tau_d}{\tau_d + \tau_w} = C_f \frac{1}{\tau_d}.$$

From the law of bubble growth (2) follows directly

$$D_d^2 f = 4C_f \beta^2. \tag{23}$$

At sufficiently large pressures or densities of the heat flux $C_f \approx 1$ and the product $D_d^2 f = 4\beta^2$ depends only on the conditions of bubble growth, i.e., pressure and temperature difference ΔT. From (18) we can obtain an equation analogous to the semiempirical MacFadden–Grassman relationship

$$D_d^{\frac{1}{2}} f = \frac{\sqrt{2}\, C_f}{C_R^{3/2}} g^{\frac{1}{2}} = C g^{\frac{1}{2}}. \tag{24}$$

The coefficient C has a maximum value in the case of the dynamic separation regime and $C_f = 1$ ($C = 0.9$), which agrees with the known experimental data [17].

From (15), (18), and (20) we obtain the following expression for the characteristic bubble velocities:

$$fD_d = 2C_f C_R^{-1} \beta^{\frac{2}{3}} g^{\frac{1}{3}} = 2C_f C_u^{-1} C_R^{-\frac{3}{2}} u; \tag{25}$$

$$\dot{S}_d = \frac{2C_R^2}{2,5+\gamma}\left[1+(1+0,7\xi)^{-\frac{5}{2}-\gamma}(0,5\xi-1)\right]\beta^{\frac{2}{3}}g^{\frac{1}{3}}. \quad (26)$$

The quantities $\dot{S}_d$ and fD_d will have maximum values in the dynamic bubble separation regime $\dot{S}_d \approx fD_dC_f^{-1} \approx 0.6u$. In the quasi-static separation regime $\dot{S}_d \approx 0$. The values of accelerations before separation ($\tau = \tau_d - 0$): $\ddot{S}_{dst} \approx 0$, $\ddot{S}_{ddyn} \approx 0.8g$.

5. Movement of Vapor Bubbles after Separation. If the bubble does not increase after separation, its equation of motion can be written with sufficient accuracy in the form [6]

$$\ddot{S}R + \frac{3}{4}C_D\dot{S}^2 - 2gR = 0. \quad (27)$$

Solving (27), we obtain

$$\dot{S} = u\frac{\exp\left[\frac{4g}{u}(\tau-\tau_d)\right]-\omega}{\exp\left[\frac{4g}{u}(\tau-\tau_d)\right]+\omega}, \quad \omega = \frac{u-\dot{S}_d}{u+\dot{S}_d}, \quad (28)$$

where the constant bubble buoyancy velocity $u \approx 2.1\sqrt{R_dg}$ for Reynolds numbers of the order of 10^2-10^3 [13].

The acceleration of the bubble at the instant of separation ($\tau = \tau_d + 0$) can be written in the form

$$\ddot{S}_d = \frac{8\omega}{(1+\omega)^2}g.$$

The values of the parameter ω lie within $\omega = 0.25$-1, whence follows $1.3g < \ddot{S}_d < 2g$, the lower limit pertaining to the dynamic and the upper to the quasi-static separation regime.

The time of the bubble's attainment of a constant buoyancy velocity after separation, determined by the 20-fold decrease of its maximum acceleration, is equal to:

$$\tau_c - \tau_d \approx \frac{3}{4}\frac{u}{g}. \quad (29)$$

The statements found in the literature that the bubble after separation immediately acquires a constant velocity [1] are true within estimates (28), (29).

NOTATION

a	is the thermal diffusivity;
D_d	is the bubble separation diameter;
F	are the forces acting on the bubble;
g	is the acceleration of gravity;
L	is the latent heat of evaporation;
R_c	is the cavity radius;
T_s	is the saturation temperature;
ΔT	is the wall—liquid temperature difference;
β	is the growth modulus;
ρ, ρ''	are the density of liquid and vapor;
μ, ν	are the dynamic and kinematic viscosity;
σ	is the surface tension;
$Ja = \lambda\Delta T/\rho''La$	is the Jacob number.

Subscripts

0	denotes the start of bubble separation;
d	denotes the bubble separation;
st	denotes the static separation regime;
dyn	denotes the dynamic separation regime.

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